Three terms and general term

e^x

sin x

cos x

ln(1+x)

1 / (1-x^2)

Feb 10-8:26 AM

18 $\sum_{n=2}^{\infty} (-1)^{n+1} \frac{1}{\ln(n)} = \frac{-1}{\ln 2} + \frac{1}{\ln 3} - \frac{1}{\ln 4} \dots$

Converge

~~diverge~~

AST

If 1. signs alternate

2. terms get smaller

3. seq $\to 0$

then series converges

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13. $\sum_{n=1}^{\infty} n \sin\left(\frac{1}{n}\right)$ diverges by n^{th} term test

n^{th} term test

$$\lim_{n\to\infty} \frac{\sin\left(\frac{1}{n}\right)}{1/n} = \lim_{n\to\infty} \frac{\cos\left(\frac{1}{n}\right)\left(\cancel{-\frac{1}{n^2}}\right)}{\left(\cancel{-\frac{1}{n^2}}\right)} = 1$$

$\frac{0}{0}$

Seq converges to 1

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9.5b Alternating Series, Checking Endpoints

Alternating Series Test with remainder AST

If

1. signs alternate

2. $|a_{n+1}| < |a_n|$ {terms get smaller}

3. $\lim_{n\to\infty} a_n = 0$

Then the series converges

$a_1 - a_2 + a_3 - a_4 \dots a_n |$

error $< |a_{n+1}|$

look at next term to estimate error

if a_{n+1} is pos, underestimate

If a_{n+1} is neg, overestimate

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Prove the alternating harmonic series is convergent but not absolutely convergent

Conditional Convergence

$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} \dots (-1)^{n+1} \frac{1}{n}$

Converges by AST

✓1. signs alt

✓2. $|a_{n+1}| < |a_n|$

✓3. $a_n \to 0$

$\sum_{n=1}^{\infty} \left|(-1)^{n+1} \frac{1}{n}\right| = 1 + \frac{1}{2} + \frac{1}{3} \dots$ diverges harmonic

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Conditional Convergence

$\sum a_n$ converges (use AST)

$\sum |a_n|$ diverges

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Find the interval of convergence for the following series. Be sure to check the endpoints.

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{2n}}{2n}$$

ioc : 3

1. interval: (ratio test) $\lim_{n\to\infty} \left| \frac{x^{2(n+1)}}{2(n+1)} \cdot \frac{2n}{x^{2n}} \right|$

$\lim_{n\to\infty} \left| x^2 \frac{2n}{2n+2} \right| = x^2 < 1$, $\frac{2n}{2n+2} \to 1$

$-1 < x \leq 1$

2. $x=1$: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 1^{2n}}{2n} = \frac{1}{2} - \frac{1}{4} + \frac{1}{6} - \frac{1}{8} \dots$

converges by AST at $x=1$

3. $x=-1$ $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(-1)^{2n}}{2n} = \frac{1}{2} - \frac{1}{4} + \frac{1}{6} - \frac{1}{8} \dots$

converges at $x=-1$ by AST

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$$\sum_{n=0}^{\infty} \frac{(10x)^n}{n!}$$ find ioc

$\lim_{n\to\infty} \left| \frac{(10x)^{n+1}}{(n+1)!} \cdot \frac{n!}{(10x)^n} \right|$

$\lim_{n\to\infty} \left| \frac{10x}{n+1} \right| = 0 < 1$ for all x

converges for all x

ioc $(-\infty, \infty)$

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$$\sum_{n=1}^{\infty} \frac{(x-3)^n}{2n}$$

1. $\lim_{n\to\infty} \left| \frac{(x-3)^{\cancel{n+1}}}{2n+2} \cdot \frac{2n}{\cancel{(x-3)^n}} \right|$

$\lim_{n\to\infty} \left| \frac{2n}{2n+2} (x-3) \right| = |x-3| < 1$, $\frac{2n}{2n+2} \to 1$

$-1 < x-3 < 1$

$2 < x < 4$

2. $x=4$ $\sum_{n=1}^{\infty} \frac{(1)^n}{2n}$ p series $p=1$ diverges

3. $x=2$ $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n} = -\frac{1}{2} + \frac{1}{4} - \frac{1}{6} \dots$ Converges by AST

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$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} \dots$

$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \dots$

let $x=1$

$\ln 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} \dots$

converge to π

$1 + \frac{1}{3} + \frac{1}{5} \dots - \frac{1}{2} - \frac{1}{4} - \frac{1}{7}$

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